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The Green function of cavity polariton with radiative correction: a new description of resonant second-order optical processes

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Abstract
A previously proposed Green function of cavity polariton associated with quantum well excitons in a distributed Bragg reflector cavity has been generalized to an arbitrary three-dimensional cavity containing multilevel excitons. This Green function describes the electromagnetic field, both inside and outside the cavity, caused by the cavity dielectrics and the linear polarization of excitons. The electromagnetic field induced by any other polarizations can be calculated as a convolution with the Green function. In this way, we can include in the Green function approach all the problems which deal with boundary conditions and the radiative shift/width of excitons.

1. Introduction

Resonant excitation of matter by light is followed by various processes, such as luminescence, resonant Raman scattering, and thermal decay. In the presence of additional exciting sources such as pump light, coherently injected phonons, and so on, even more complicated phenomena can arise. There is much current interest in observing these phenomena in a microcavity, which provides us with a peculiar mode and spatial electromagnetic (EM) field structure. Namely, if the cavity confinement is good enough, we have well defined cavity modes with a discrete spectrum, and each mode has a characteristic spatial structure. Using this feature, we can realize particular cases of radiation matter interaction, such as parametric amplification of cavity polariton [1].

A common feature of these phenomena is that the resonant excitation drives the coupled radiation–matter system dynamically, where the evolution is caused either coherently or incoherently. The coherent processes will appear in the nonlinear optical processes or those induced by coherent phonons, and the incoherent ones in the thermal processes and radiative decay.

The radiative decay is an important factor in a microcavity, since it represents the fact that the cavity modes and the external EM field are connected. In the absence of radiative decay, external light cannot excite matter in the cavity, and cavity polariton cannot produce signal light outside the cavity. Concerning this point, there is a problem about its theoretical treatment in literature, i.e. the so-called 'quasi-mode coupling' [2].

In the scheme of quasi-mode coupling, one first supposes that the cavity modes are completely confined, i.e. that they have infinitely large Q-factors. The dynamical processes of the excited levels of the matter in the cavity are supposed to occur only with these cavity modes. But one allows the interaction of these coupled cavity polariton with external EM field, in the sense that the excitation and observation of the cavity polariton via external photons become possible. The coupling of a cavity mode with external light is introduced so that the experimental values of the radiative width of the cavity mode is reproduced.

The introduction of the interaction between the cavity and external modes cannot be justified from the first-principles of EM theory. If one completely confines cavity modes, for example by using a mirror of 100% reflectivity, both the cavity and external modes have zero amplitude at the cavity walls. Then, the macroscopic EM theory allows these amplitudes to have independent values, so that no decay is expected from this scheme.

It is a correct boundary condition, and not the interaction, that converts the ill-defined modes to correct ones. The physical picture of a finite Q-valued cavity mode is that it consists of EM field components both inside and outside the cavity. The modes of the external field are usually defined in terms of a large size box ($\rightarrow \infty$ finally), leading to a continuum spectrum. In principle, any mode of this continuum has amplitude both inside and outside the cavity. Those having larger amplitudes inside, or outside, the cavity are the cavity, or external, modes respectively. By changing the eigenfrequency, a mode evolves smoothly between the cavity and external modes, and there is no sharp boundary between them except for the case when $Q \rightarrow \infty$. Thus, a reasonable treatment of finite Q-valued cavity modes should be based on the correct treatment of the boundary conditions of the detailed cavity structure.

With the above-mentioned criticism, the present author introduced a new EM Green function of cavity polariton, describing the EM field both inside/outside the microcavity and explicitly taking the incomplete confinement of EM field by a microcavity [3]. In this Green function, both of the EM fields induced by (a) the background polarization of the cavity and (b) the linear polarization of quantum well exciton are taken into account in an analytical form. The Green function has poles at the frequencies of the external photon modes disturbed by the cavity structure, among which leaky cavity modes are included. In the limit of an infinitely large box one needs to define the external modes, the cavity mode is described by a complex eigenfrequency, the imaginary part of which corresponds to the radiative width of the cavity mode. The merit of this Green function is that the additional EM field caused by other sources such as nonlinear processes and phonon induced processes can be easily obtained as a convolution of the Green function and the polarization induced by the additional sources.

An analytic expression of the Green function mentioned above is obtained according to a special model of one-dimensional (1D) confinement and the assumption of a single resonant level. Though it is very simple, it can be applied to a quantum well exciton in a distributed Bragg reflector (DBR) cavity. However, it is quite desirable to generalize it to less restrictive models. In this paper, we give the result of such generalization, i.e. we derive an analytic form of the Green function for a matter system with three-dimensional (3D) confinement and with an arbitrary number of resonant levels.

In the discussion, we will mention various possibilities of application of this type of Green function, especially from the viewpoint of a 'new type of description of resonant second (and higher) order optical processes'.

2. Green function of cavity polariton: the general case

Any matter system consists of many levels of excited states. Even if an incident light field is resonant with one of them, all the excited states respond to it with various weights. Since the number of excited states is infinite, it is not possible to take all of them into account as dynamical variables in considering light–matter interaction. The usual solution for this problem is to separate the induced polarization into resonant and non-resonant parts according to their eigenfrequencies, and treat the non-resonant part as background dielectrics. In this way, the quantum mechanical motion of the resonant components is treated as explicit dynamical variables, while the remaining degrees of freedom are not just thrown away but treated as a locally polarizable medium.

If the background dielectrics are arranged in an appropriate way, they will constitute a cavity, where the mode structure of the EM field is strongly changed from that in vacuum. In ideal cavities, the EM field can exist only for a series of discrete frequencies, which are obtained from the non-escape condition imposed on the solutions of the Maxwell equations in the cavity. In reality, all the cavity modes leak out of the cavity, and all the external EM modes can penetrate the cavity. In this sense, there is no large difference between a cavity structure and other dielectrics. Though we know some typical examples of cavity structures, such as DBR cavities and dielectric sphere with whispering gallery modes, the Q-values of their cavity modes are not necessarily high. Generally speaking, the mode and spatial structure of the EM field change in the presence of a matter, and if the deposition place and electronic structure of the matter are appropriate, the change is such that well-defined cavity modes arise. In this sense, we can regard the background part of induced polarization mentioned above as a (good or bad) cavity.

In this section, we consider the general case of a 3D optical medium consisting of multiple resonant levels and background dielectrics representing non-resonant polarizations.

Let us write the Maxwell equations, by eliminating the magnetic field, as

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, \omega) - q^2\mathbf{E}(\mathbf{r}, \omega) = 4\pi q^2\mathbf{P}(\mathbf{r}, \omega), \tag{1}$$

where ω is frequency, $q = \omega/c$, and $\mathbf{E}$ and $\mathbf{P}$ are the electric field and polarization, respectively. We now divide $\mathbf{P}$ into three parts as

$$\mathbf{P} = \mathbf{P}_\mathrm{b} + \mathbf{P}_\mathrm{x}^{(1)} + \mathbf{P}', \tag{2}$$

where $\mathbf{P}_\mathrm{b}$ is the background part of the polarization, $\mathbf{P}_\mathrm{x}^{(1)}$ the linear polarization of resonant levels of the matter, and $\mathbf{P}'$ the remaining part of the induced polarization.

The background part $\mathbf{P}_\mathrm{b}$ is assumed to be written as

$$\mathbf{P}_\mathrm{b}(\mathbf{r}, \omega) = \chi_\mathrm{b}(\mathbf{r})\mathbf{E}(\mathbf{r}, \omega) \tag{3}$$

in terms of background susceptibility χ_b. The susceptibility χ_b has no microscopic position dependence, because it represents a macroscopic non-resonant response, but it may have a macroscopic position dependence, if the background dielectrics have an internal structure such as DBR. Including $\chi_\mathrm{b}(\mathbf{r}) = 0$ for $\mathbf{r}$ outside the background medium, the above equation holds over all the space. As the values of χ_b, we can use the bulk values corresponding to the structure of the background part of the matter.

The contribution of the resonant levels $\mathbf{P}_\mathrm{x}^{(1)}$ should be treated quantum mechanically. Namely, its relationship with the source electric field is given in a nonlocal form as [4]

$$\mathbf{P}_\mathrm{x}^{(1)}(\mathbf{r}, \omega) = \sum_\mu \frac{\mathbf{P}_{0\mu}(\mathbf{r})}{(E_{\mu 0} - \hbar\omega - \mathrm{i}\gamma)} \int \mathrm{d}\mathbf{r}'\, \mathbf{P}_{\mu 0}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}', \omega). \tag{4}$$

Here, $(\mu, 0)$ are the quantum numbers of the excited and ground states, respectively, $\mathbf{P}_{0\mu}(\mathbf{r})$ is the matrix element of polarization (electric dipole density) operator, and γ is the non-radiative damping parameter introduced phenomenologically. For quantum well (QW) structures, one can think that the nonlocal form is not necessary. However, the nonlocal form is not only correct from first principles, but also convenient for the purpose of solving the integro-differential equation for the Green function, as seen below.

The explicit form of $\mathbf{P}'$ should be considered according to the problem to be solved. Later, we will give an example for the case of acoustic phonon induced polarization.

The Green functions to solve equation (1) can be considered in various ways. If we use the Green function for vacuum $\mathbf{G}_{\rm vac}(\mathbf{r}, \mathbf{r}', \omega)$ defined as

$$\nabla \times \nabla \times \mathbf{G}_{\rm vac} - q^2 \mathbf{G}_{\rm vac} = 4\pi q^2 \delta(\mathbf{r} - \mathbf{r}'), \tag{5}$$

we obtain the solution of equation (1) as

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_0^{(\rm vac)}(\mathbf{r}, \omega) + \int {\rm d}\mathbf{r}'\, \mathbf{G}_{\rm vac}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{P}(\mathbf{r}', \omega). \tag{6}$$

The solution of the homogeneous equation, $\mathbf{E}_0^{(\rm vac)}$, represents the incident field in vacuum, and the integral part gives the polarization induced field. Since the Green function describes the propagation of the vector field, it is a tensor. The analytic expression of this Green function can be derived easily as [4]

$$\mathbf{G}_{(\rm vac)}(\mathbf{r}, \mathbf{r}') = \left[q^2 \mathbf{1} + \nabla\nabla\right] \frac{\exp[{\rm i}q|\mathbf{r} - \mathbf{r}'|]}{|\mathbf{r} - \mathbf{r}'|}. \tag{7}$$

Another Green function can be defined by renormalizing the background part of polarization, which represents the EM field inside and outside the background dielectrics without other polarization sources. This Green function $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$ should satisfy

$$\nabla \times \nabla \times \mathbf{g} - q^2 \epsilon_{\rm b}(\mathbf{r}) \mathbf{g} = 4\pi q^2 \delta(\mathbf{r} - \mathbf{r}'), \tag{8}$$

where $\epsilon_{\rm b}(\mathbf{r}) = 1 + 4\pi \chi_{\rm b}(\mathbf{r})$. The explicit form of this Green function depends on the construction of the background dielectrics of the system under consideration. For multilayer planar and spherical structures it is easy to obtain an analytic form of $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$ [5]. In terms of this Green function, the solution of equation (1) is given by

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_0^{(\rm cav)}(\mathbf{r}, \omega) + \int {\rm d}\mathbf{r}'\, \mathbf{g}(\mathbf{r}, \mathbf{r}') \cdot \left[\mathbf{P}_{\rm x}^{(1)}(\mathbf{r}', \omega) + \mathbf{P}'(\mathbf{r}', \omega)\right]. \tag{9}$$

Note that the polarization source in the integrand consists of $\mathbf{P}_{\rm x}^{(1)}$ and $\mathbf{P}'$, and that the incident field $\mathbf{E}_0^{(\rm cav)}$ includes the effect of multiple scattering due to background dielectrics.

We now proceed to include the linear polarization $\mathbf{P}_{\rm x}^{(1)}$ into the Green function, which we call the Green function of cavity polariton $\mathbf{G}_{\rm cp}(\mathbf{r}, \mathbf{r}', \omega)$. Its definition is

$$\nabla \times \nabla \times \mathbf{G}_{\rm cp} - q^2 \epsilon_{\rm b}(\mathbf{r}) \mathbf{G}_{\rm cp} - \sum_{\mu} \frac{4\pi q^2 \mathbf{P}_{0\mu}(\mathbf{r})}{(E_{\mu 0} - \hbar\omega - {\rm i}\gamma)} \mathbf{H}_{\mu 0}(\mathbf{r}', \omega) = 4\pi q^2 \delta(\mathbf{r} - \mathbf{r}'), \tag{10}$$

where

$$\mathbf{H}_{\mu 0}(\mathbf{r}', \omega) = \int {\rm d}\mathbf{r}''\, \mathbf{P}_{\mu 0}(\mathbf{r}'') \cdot \mathbf{G}_{\rm cp}(\mathbf{r}'', \mathbf{r}', \omega). \tag{11}$$

We note the identity for an arbitrary vector function $\mathbf{F}$ [3]

$$\left[-q^2 \epsilon_{\rm b}(\mathbf{r}) + \nabla \times \nabla \times\right]^{-1} \mathbf{F}(\mathbf{r}) = \frac{1}{4\pi q^2} \int {\rm d}\mathbf{r}'\, \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{F}(\mathbf{r}'), \tag{12}$$

which can easily by proved by inverting the inverse operator on the left-hand side and using the definition of $\mathbf{g}$. This allows us to get the solution of equation (10) as

$$\mathbf{G}_{\rm cp}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) + \sum_{\nu} \frac{\mathbf{h}_{0\nu}(\mathbf{r}, \omega)}{(E_{\nu 0} - \hbar\omega - {\rm i}\gamma)} \mathbf{H}_{\nu 0}(\mathbf{r}', \omega), \tag{13}$$

where the electric field induced by $\mathbf{P}_{0\nu}$ in the empty cavity is defined as

$$\mathbf{h}_{0\nu}(\mathbf{r}) = \int \mathrm{d}\mathbf{r}'\, \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{P}_{0\nu}(\mathbf{r}'). \tag{14}$$

Substituting equation (13) into (11), we obtain a set of linear equations for $\{\mathbf{H}_{\mu 0}\}$ as

$$\mathbf{H}_{\mu 0}(\mathbf{r}, \omega) = \mathbf{h}_{\mu 0}(\mathbf{r}, \omega) - \sum_{\nu} \frac{B_{\mu\nu}(\omega)}{E_{\nu 0} - \hbar\omega - \mathrm{i}\gamma} \mathbf{H}_{\nu 0}(\mathbf{r}, \omega), \tag{15}$$

where the interaction between two components of induced polarization via the cavity EM field (radiative correction) is defined as

$$B_{\mu\nu}(\omega) = -\int\int \mathrm{d}\mathbf{r}\, \mathrm{d}\mathbf{r}'\, \mathbf{P}_{\mu 0}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{P}_{0\nu}(\mathbf{r}'). \tag{16}$$

This quantity can be interpreted in terms of the interaction energy between the polarization $\mathbf{P}_{\mu 0}$ and the electric field induced by the polarization $\mathbf{P}_{0\nu}$ in the background dielectrics. We can also introduce the electric field induced by the polarization $\mathbf{P}_{\mu 0}$ as

$$\mathbf{h}_{\mu 0}(\mathbf{r}', \omega) = \int \mathrm{d}\mathbf{r}'\, \mathbf{P}_{\mu 0}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega). \tag{17}$$

The solution of equation (15) is obtained as

$$\mathbf{H}_{\mu 0}(\mathbf{r}, \omega) = \sum_{\nu} [\mathbf{1} + \mathbf{C}]^{-1}_{\mu\nu} \mathbf{h}_{\nu 0}(\mathbf{r}, \omega), \tag{18}$$

where

$$C_{\mu\nu} = \frac{B_{\mu\nu}(\omega)}{E_{\nu 0} - \hbar\omega - \mathrm{i}\gamma}. \tag{19}$$

Substituting equation (18) into (13), we obtain the analytic expression of $\mathbf{G}_{\mathrm{cp}}$ as

$$\mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) + \sum_{\mu}\sum_{\nu} \mathbf{h}_{0\mu}(\mathbf{r}, \omega) \frac{[\mathbf{1} + \mathbf{C}]^{-1}_{\mu\nu}}{E_{\mu 0} - \hbar\omega - \mathrm{i}\gamma} \mathbf{h}_{\nu 0}(\mathbf{r}', \omega). \tag{20}$$

This is the desired expression of the Green function of cavity polariton, which gives the solution of equation (1) as

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_0^{(\mathrm{cp})}(\mathbf{r}, \omega) + \int \mathrm{d}\mathbf{r}'\, \mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{P}'(\mathbf{r}', \omega). \tag{21}$$

Here, the incident field $\mathbf{E}_0^{(\mathrm{cp})}$ contains the effects of multiple scattering from both the background dielectrics and the resonant polarization arising from QW excitons.

If we can neglect the off-diagonal element of $B_{\mu\nu}$, the Green function of cavity polariton takes a simplified form of

$$\mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega) \simeq \mathbf{g}(\mathbf{r}, \mathbf{r}', \omega) + \sum_{\mu} \frac{\mathbf{h}_{0\mu}(\mathbf{r}, \omega)\mathbf{h}_{\mu 0}(\mathbf{r}', \omega)}{E_{\mu 0} + B_{\mu\mu}(\omega) - \hbar\omega - \mathrm{i}\gamma}. \tag{22}$$

If we keep just one term of μ in the above expression, it essentially reproduces the result of [3].

3. Discussion

3.1. Characteristic feature of the obtained results

In the previous section we have shown that a knowledge of the Green function $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$ for the background dielectrics (or cavity) leads to an analytic expression of the Green function $\mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega)$ of cavity polariton, and that, in terms of this Green function $\mathbf{G}_{\mathrm{cp}}$, the field due to any other source of polarization can be written explicitly in the form of a convolution of this Green function and the additional polarization.

This can be done by using the quantum mechanical expression of resonant polarization with nonlocal relationship to the source field. The nonlocal susceptibility in equation (4) is

$$\chi_{\mathrm{x}}^{(1)}(\mathbf{r}, \mathbf{r}', \omega) = \sum_{\mu} \frac{\mathbf{P}_{0\mu}(\mathbf{r})\mathbf{P}_{\mu 0}(\mathbf{r}')}{E_{\mu 0} - \hbar\omega - \mathrm{i}\gamma}, \tag{23}$$

which plays the role of an integral kernel to define the induced polarization. The remarkable point is that it generally has a separable form [3], and because of this, the Green function of cavity polariton has been obtained analytically.

As to the Green function of cavity, $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$, we have to specify the construction of the background dielectrics of the problem under consideration. For planar or spherical multilayer structures, it is possible to calculate $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$ in an analytical form [5]. For other cases, we need a numerical method. In any case, we know the role of $\mathbf{g}(\mathbf{r}, \mathbf{r}', \omega)$ from the expression for $\mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega)$: it defines the induced field and the radiative interaction energy, so that appropriate treatment in numerical calculation would be possible.

It should be noted that the poles of a Green function reflect the medium which determines the EM field one needs to include in the Green function. In the case of $\mathbf{G}_{\mathrm{vac}}$, its poles correspond to the frequencies of vacuum photons associated with the large size box quantization, and as the box size goes to infinity, they tend to form a continuum. The poles of $\mathbf{g}$ are shifted from those of $\mathbf{G}_{\mathrm{vac}}$ due to the presence of the background dielectrics, together with the corresponding change in the spatial structure of each mode. If the background dielectrics represent a cavity, the frequency shift and the change in spatial pattern are such that for a group of frequencies (or bands) the modes have main amplitudes in the cavity and the rest outside the cavity. The boundary between the two groups is not sharp (except for the case of 100% confinement), so that the character of a mode changes smoothly from one to the other as we sweep frequency.

The poles of $\mathbf{G}_{\mathrm{cp}}$ in the case of a cavity structure contain the coupled mode frequencies, e.g., those of cavity polariton in the case of a quantum well exciton in a DBR cavity, and they generally show anti-crossing behaviour at the crossing points of the dispersion curves of bare cavity modes and bare electronic excitations. The poles of the external modes are influenced by the cavity polariton, together with the corresponding change in their spatial structure.

In the case of a multilayer cavity with 1D confinement, another type of poles for $\mathbf{g}$ can become important. When we consider lateral wavevectors in a wide range, different cavity modes can contribute. For a larger lateral wavevector, a cavity mode with more nodes becomes possible for a given frequency. This effect is automatically taken into account by the use of $\mathbf{g}(z, z', \mathbf{k}_{\|}, \omega)$, according to the microscopic construction of the cavity.

It should be stressed that the above scheme takes, from first principles, proper account of the radiative damping and shift of the matter excited states. It provides us with a new approach to the description of resonant second-order optical processes, since it enables us to treat radiative decay in a system-dependent manner. Since the radiative decay of the matter resonance is already included in the Green function $\mathbf{G}_{\mathrm{cp}}$ within the scheme of linear response, we only need to describe $\mathbf{P}'$ for the second order process to be considered. In the cases of photoluminescence and Raman scattering, $\mathbf{P}'$ is the induced polarization by phonons in photo-excited systems.

3.2. Radiative width of an additional excitation

The strong modification of the EM field by the cavity polariton system will influence the radiative behaviour of an additional excitation, which is not included in $\mathbf{P}_{\mathrm{x}}^{(1)}$. As such an excitation, we can consider a two-level excitation placed inside or in the close neighbourhood of the cavity polariton system. If the additional excitation to be considered has a resonant energy E_1 and the matrix element of polarization $\mathbf{P}_1(\mathbf{r})$, we can describe the dynamical motion of this additional excitation interacting with the EM field of the cavity polariton according to (the revised form of) the nonlocal response theory [6]. The equation to determine the amplitude X of the induced polarization self-consistently is

$$\left[E_1 - \hbar\omega - \mathrm{i}\gamma - \int\int \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'\,\mathbf{P}_1(\mathbf{r})^* \cdot \mathbf{G}_{\mathrm{cp}}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{P}_1(\mathbf{r}')\right] X = F_1^{(0)}(\omega), \tag{24}$$

where $F_1^{(0)}(\omega)$ represents the interaction of the additional mode with an incident field. Due to the assumption of a single additional excitation, this is a linear equation for one variable X (if one considers N additional excitations, the simultaneous linear equations for N variables $\{X\}$ need to be considered). The eigenmode of the coupled system is determined by the condition of non-trivial solution ($X \neq 0$ for vanishing external field), i.e. the vanishing coefficient of the above equation. The roots ω are complex numbers in general. Noting that $\mathbf{G}_{\mathrm{cp}}$ has poles at the cavity polariton frequency, the eigenmodes of this equation have the mixed character of the E_1 excitation and the cavity polariton. Through the mixing, the E_1 excitation is modified to have a shift and a width, and this width is the radiative width caused by the EM field represented by $\mathbf{G}_{\mathrm{cp}}$. The width of the cavity polariton due to the leakage from the cavity is shared by the new coupled modes. It should also be noted that this 'radiative' correction contains, in addition to the pure radiative shift, the shift due to the coupling of $\mathbf{P}_1$ with the longitudinal electric field contained in the tensor Green function $\mathbf{G}_{\mathrm{cp}}$. This is a further generalization of the coupling of a two-level system with a whispering gallery mode of a dielectric sphere discussed in [7], which can be treated by equation (24) with $\mathbf{G}_{\mathrm{cp}}$ replaced by $\mathbf{g}$.

3.3. Possible applications

The Green function of cavity polariton can be used for various problems. By specifying the additional induced polarization $\mathbf{P}'$, we can describe many different cases. All the dynamical processes driven from the initially given states of $\mathbf{P}_{\mathrm{x}}^{(1)}$ by thermal phonons, an externally induced coherent phonon wave, pump light, etc will be appropriately treated with the Green function. What is needed is to specify the form of $\mathbf{P}'$ for each problem.

As an example, we give some explicit expressions for the case of the QW excitons in a DBR cavity driven by an externally applied surface acoustic wave of a given wavevector $\mathbf{k}_\parallel$ and frequency Ω. This is a kind of 'dynamical Bragg scattering effect' for cavity polariton. Due to the presence of a surface acoustic wave, the motion of the exciton polarization induced by an incident light (with wavevector $\mathbf{q}_\parallel$ and frequency ω) will be modulated by the acoustic phonon. This produces various components of electric field and polarization with $\mathbf{q}_\parallel + n\mathbf{k}_\parallel$ with the corresponding frequencies $\omega + n\Omega$, where $n = 0, \pm1, \pm2, \ldots$ [8].

Solving the equation of motion for polarization, we can relate the Fourier components of the electric field and polarization in terms of acoustic phonon induced susceptibilities as

$$P_n'(z) = \sum_\ell \chi_{n,\ell}\, E_\ell(z) \tag{25}$$

where $P_n'(z) = P'(z, \mathbf{q}_\parallel + n\mathbf{k}_\parallel, \omega + n\Omega)$, $E_n(z) = E(z, \mathbf{q}_\parallel + n\mathbf{k}_\parallel, \omega + n\Omega)$ and $\chi_{n,\ell}$ is the susceptibility relating these components of polarization and electric field. In the diagonal element $\{\chi_{n,n}\}$, there are contributions from the pure electronic part and the phonon induced part. Since the electronic part represents the QW exciton, we subtract it from $\chi_{n,n}$. Then $P_n'(z)$ represents purely phonon induced polarization.

Now, the electric field produced by $P'_n(z)$ is given via $\mathbf{G}_{\mathrm{cp}}$ as

$$E_n(z) = E_n^{(0)}(z) + \int \mathrm{d}z'\, \mathbf{G}_{\mathrm{cp}}(z, z', \mathbf{q}_\| + n\mathbf{k}_\|, \omega + n\Omega) P'_n(z') \tag{26}$$

Substituting equation (25) into (26), we have a set of self-consistent equations to determine $\{E_n(z)\}$ for z inside the QW and for an incident field $E^{(0)}_{n=0}(z)$. Once we obtain the solutions $\{E_n(z)\}$, we can determine $\{P'_n(z)\}$, which allows us to calculate $\{E_n(z)\}$ at arbitrary z, i.e. the signal amplitudes. Then, the self-consistently determined field is given by equation (26), which contains various 'dynamically Bragg scattered terms'. Such terms will be enhanced, when the acoustic phonon of given $\mathbf{k}_\|$ and Ω connects two points on cavity polariton branches [9].

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